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SOME RESULTS ON CUBIC HARMONIOUS LABELING

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Abstract

A *(n,m)* graph $G = (V,E)$ is said to be **Cubic Harmonious Graph(CHG)** if there exists an injective function $f: V(G) \rightarrow \{1,2,3,\ldots\ldots m^3+1\}$ such that the induced mapping $f^*_{chg}: E(G) \rightarrow \{1^3, 2^3, 3^3, \ldots\ldots m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. Here we will discuss about cubic harmonious labeling techniques of the graph $P_n \Theta K_1 - e$ and twig graph.

Keywords:

Comb graph, Cubic harmonious graph, Harmonious graph, Path graph. Twig graph.

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1. Introduction

By a graph, we mean a finite, undirected graph without loops or multiple edges. A path on n vertices is denoted by P_n. Graph labeling, where the vertices are assigned certain values subject to some conditions, have often motivated by practical problems. In the last five decades, enormous work has been done on this subject [3].Any graph labeling will have the following three common characteristics :
(1). A set of numbers from which the vertex labels are chosen.
(2). A rule that assigns a value to each edge.
(3). A condition that these values must satisfy.
Graph labelings is an active area of research in graph theory which is mainly evolved through its rigorous applications in coding theory, communication networks, optional circuits layouts and graph decomposition problems. According to Beineke and Hegde [2] graph labeling serves as a frontier between number theory and structure of graphs. For a dynamic survey of various graph labeling problems along with an extensive bibliography we refer to Gallian [4]. Graham and Sloane[5] introduced harmonious labeling . Square harmonious graphs were introduced in [10]. Cubic graceful graphs were introduced in [6]. Cubic harmonious graphs were defined in [7].

Definition 1.1
The ***path*** on *n* vertices is denoted by P_n.
Definition1.2
The ***Twig graph*** obtained from the the path P_n by attaching exactly two pendant edges to each internal vertex of the path .
Definition1.4

Let $P_n \Theta K_1$ be the ***Comb*** which is the graph obtained from a path P_n by attaching pendant edge at each vertex of the path .

II.MAIN RESULTS

Theorem 2. 1

The graph $P_n \Theta K_1 - e$ is cubic graceful for all $n \geq 1$.

Proof:

Let G be the graph $P_n \Theta K_1 - e$.

Let $V(P_n \Theta K_1 - e) = \begin{cases} u_i, & 1 \leq i \leq n \\ v_i, & 1 \leq i \leq n-1 \end{cases}$

And

$E(P_n \Theta K_1 - e) = \begin{cases} u_i u_{i+1} \; ; & 1 \leq i \leq n-1 \\ u_i v_i \; ; & 1 \leq i \leq n-1 \end{cases}$

Then $|V(P_n \Theta K_1 - e)| = 2n-1$ and $|E(P_n \Theta K_1 - e)| = 2n-2$

Define an injection $f : V(P_n \Theta K_1 - e) \rightarrow \{1,2,3,\ldots\ldots,(2n-2)^3+1\}$ by

$$f(u_i) = (2n-2)^3 + 1$$
$$f(u_i) = (2n-i)^3 + (2n-2)^3 + 1 - f(u_{i-1}) \; ; \quad 2 \leq i \leq n$$
$$f(v_i) = (2n-2)^3 + 1 + i^3 - f(u_i) \; ; \quad 1 \leq i \leq n-1$$

The induced edge mapping are

$$f^*(u_i u_{i+1}) = (2n-1-i)^3; \quad 1 \leq i \leq n-1$$
$$f^*(u_i v_i) = i^3 \; ; \quad 1 \leq i \leq n-1$$

The vertex labels are in the set $\{1,2,3,\ldots\ldots, (2n-2)^3+1\}$. Then the edge labels are distinct and cubic . They are $\{1^3,2^3,3^3,\ldots\ldots\ldots,(2n-2)^3\}$.Hence the theorem.

Theorem 2.2

Let P_n be the path on vertices. Then the twig graph G obtained from the path P_n by attaching exactly two pendant edges to each external vertex of the path is cubic harmonious.

Proof:

Let G be the twig graph.

Let $V(G) = \begin{cases} v_i \; ; & 1 \leq i \leq n \\ u_j, w_j \; ; & 2 \leq j \leq n-1 \end{cases}$

and

$E(G) = \begin{cases} v_i v_{i+1} \; ; & 1 \leq i \leq n-1 \\ v_j u_j, v_j w_j ; & 2 \leq j \leq n-1 \end{cases}$

Then

$|V(G)| = 3n-4$ and $|E(G)| = 3n-5$

Let $f : V(G) \rightarrow \{1,2,3,\ldots\ldots\ldots\ldots,(3n-5)^3+1\}$ be defined as follows.

$$f(v_1) = (3n-5)^3 + 1$$
$$f(v_i) = (3n-3-i)^3 + (3n-5)^3 + 1 - fv_{i-1} \; ; \quad 2 \leq i \leq n$$
$$f(u_i) = (2n-2i)^3 + (3n-5)^3 + 1 - f(v_i) \; ; \quad 2 \leq i \leq n-1$$
$$f(w_i) = (2n-1-2i)^3 + (3n-5)^3 + 1 - f(v_i) \; ; \quad 2 \leq i \leq n-1$$

Let f^* be the induced edge labeling of f . Then

$$f^*(v_i v_{i+1}) = (3n-4-i)^3 \; ; \quad 1 \leq i \leq n-1$$

$$f^*(v_i u_i) = (2n-2i)^3 \; ; \quad 2 \leq i \leq n-1$$

$$f^*(v_i w_i) = (2n-1-2i)^3 \; ; \quad 2 \leq i \leq n-1$$

The vertex label are in the set $\{1, 2, \ldots\ldots, (3n-5)^3+1\}$. Then the edge labels are distinct and cubic. The edge sets are $\{1^3, 2^3, 3^3 \ldots\ldots, (3n-5)^3\}$. Hence the twig graph is cubic harmonious graph.

III.CONCLUSION

The harmonious labeling is one of the most important labeling techniques. As all the graphs are not harmonious, it is very interesting to investigate graphs or graph families which admit harmonious labeling.We have reported the cubic harmonious labeling of different graphs.

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